CONDITIONS NEAR THE ELECTRODE IN A PLASMA WITH AN ALKALI ADMIXTURE

V. N. Mikhailov

UDC 537.529

An analysis is made of the effects of certain processes in the interior of the gas and at the electrode surface on the potential drop near the electrode in a discharge in a dense, slightly ionized gas. Thermionic emission from the electrode, the Schottky effect, diffusion, and volume and surface ionization and recombination are taken into account. The analysis is carried out for a simple discharge-gap geometry: two infinite, plane-parallel electrodes. Relations are found for the potential drop near the electrode in a two-temperature plasma as a function of the discharge parameters and emission characteristics of the material. The calculated results are compared with experiment.

Processes occurring near electrodes in plasma containing an alkali admixture have recently come under intense study in connection with the development of methods for direct conversion of thermal energy [1, 2]. To a large extent, this interest results from the study of MHD conversion at small prototype installations, for which the potential behavior near the electrodes is of considerable importance. There are other reasons for the interest in these electrode processes: such important considerations as electrode erosion and the stability of a distributed discharge are closely associated with processes occurring near the electrodes [3, 4].

We assume that the plasma consists of a mixture of a gas having a high ionization potential, at a quite high pressure ($p_a \sim 10^{-2}$–10^{-4} abs. atm), and the vapor of an alkali metal ($p_g \sim 1$ abs. atm), a small part of which is ionized. We assume the gas parameters and emission characteristics of the electrodes to be known, and we seek the electrical characteristics of the discharge and the potential drop near the electrodes.

In a dense, slightly ionized gas the motion of charged particles everywhere except within a distance on the order of the mean free path from the electrodes is governed by diffusion and mobility. The electron and ion fluxes are described by [5]

$$\begin{aligned} \mathbf{f}_e &= n_e \mathbf{v}_e = -(D_e \nabla n_e + \mu_e n_e \mathbf{E}) \\ \mathbf{f}_i &= n_i \mathbf{v}_i = -(D_i \nabla n_i - \mu_i n_i \mathbf{E}) \end{aligned} \tag{1}$$

where D_e and D_i are the diffusion coefficients, μ_e, and μ_i are the mobilities, the indices "e" and "i" refer to electrons and ions, respectively, and the rest of the notation is standard. The diffusion coefficients and mobilities are related by the Einstein relations

$$\mu_e = \frac{eD_e}{kT_e}, \qquad \mu_i = \frac{eD_i}{kT_i} = \frac{eD_i}{kT_g}$$

The ion temperature is assumed equal to the temperature of the neutral particles and to the electrode temperature: $T_i = T_g = T_w$.

We write the continuity equations for the plasma components in standard form [6, 7],

$$\nabla \cdot \mathbf{f}_e = \nabla \cdot \mathbf{f}_i = \beta n_a n_e - \alpha n_i n_e^2 \tag{2}$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 3-12, July-August, 1971. Original article submitted February 7, 1971.

where β and α are the ionization and recombination coefficients, and n_a is the concentration of impurity atoms.

System (1), (2) must be supplemented with the Maxwell equations for the electric field and a relation for the current density:

$$\nabla \cdot \mathbf{E} = \frac{e}{\varepsilon_0}(n_i - n_e), \qquad \nabla \times \mathbf{E} = 0$$
$$j = j_i + j_e = e(f_i - f_e) \tag{3}$$

With the appropriate boundary conditions specified, system (1)-(3) gives a complete description of the problem if the ionization and recombination coefficients and the dependence of T_e on the other discharge parameters are known.

To determine T_e and β we can use the approximate model of a two-temperature plasma [8, 9], which is based on the following two assumptions:

1) In the unperturbed plasma* there is an ionization equilibrium at the electron temperature T_e, so the charged-particle density can be calculated from the Saha equation

$$n_{e\infty}^2 = n_a K(T_e) = n_a \frac{(2\pi m_e k T_e)^{3/2}}{h^3} \exp\left(-\frac{e\varphi_i}{kT_e}\right) \tag{4}$$

2) The electron temperature is determined from the energy-balance equation for the electron gas, written as

$$\frac{j^2}{\sigma} = \left[\sum_k \delta \left(\frac{m_e}{m_k}\right) \nu_{ek}\right] n_{e\infty} \frac{3}{2} k (T_e - T_g) \qquad (\sigma = e\mu_e n_{e\infty}) \tag{5}$$

Here $K(T_e)$ is the Saha equilibrium constant, φ_i is the impurity ionization potential, σ is the electrical conductivity of the plasma, ν_{ek} is the frequency of collisions with atoms of species k, δ is a parameter characterizing the energy loss of electron gas, and the rest of the notation is standard.

Using the assumption that an ionization equilibrium exists at the electron temperature, and using the principle of detailed equilibrium, we can determine from Eq. (2) the ionization coefficient as a function of the recombination coefficient and the equilibrium constant:

$$\beta = \alpha K(T_e) = \alpha n_{e\infty}^2 / n_a \tag{6}$$

Below we assume that for a given impurity species the quantity α and thus β depend on only the electron temperature. Under these assumptions, the α value can be taken from [6]. A more rigorous approach to the determination of the ionization and recombination coefficients has been reported elsewhere [7].

We assume that the electron temperature T_e is spatially uniform (for a given current density) because of the high electronic thermal conductivity, to within a distance from the electron on the order of the mean free path of the charged particles. Then the diffusion coefficient and mobility D_e and μ_e are also constant.

The perturbed-plasma region near the hot ($T_W \sim 2000°K$) electrode in a dense, slightly ionized gas can be divided into an ambipolar-diffusion region ($n_e = n_i = n$) and a space-charge sheath ($n_i = n_e$) [10].

In the ambipolar-diffusion region system (1)-(3) can be solved analytically; the solution is

$$j_e = \frac{\mp j_{i*}(1 - N^2) + j}{1 + \varepsilon}, \qquad j_i = \frac{\pm j_{i*}(1 - N^2) + \varepsilon j}{1 + \varepsilon} \tag{7}$$

$$E = \frac{\pm \theta(1 - \varepsilon/\theta) j_{i*}}{\varepsilon\sigma(1 + \theta)(1 + \varepsilon)} \frac{1 - N^2}{N} + \frac{j}{\sigma(1 + \varepsilon)N} \tag{8}$$

$$N = \mathrm{th}\left[\mathrm{Arth}\, N_s + \frac{j_{i*} e / kT_g}{\varepsilon\sigma(1 + \theta)} x\right] \tag{9}$$

Here and below the upper sign refers to the negatively charged electrode (the cathode), and the lower sign corresponds to the anode; the subscript "s" denotes the value of a quantity at the boundary between the ambipolar-diffusion region and the space-charged sheath, and the coordinate x is reckoned from this boundary:

*Here "unperturbed plasma" refers to that part of the discharge in which there are no gradients in the charged-particle densities.

$$j_{i*} = \frac{\sqrt{2}}{2} eD_i^{1/2}(1+\theta)^{1/2}(1+\varepsilon)^{1/2}\alpha^{1/2}n_{e\infty}^2$$

$$\varepsilon = \mu_i/\mu_e, \quad \theta = T_e/T_g, \quad N = n/n_{e\infty}.$$

The potential change in the ambipolar-diffusion region is

$$\Delta V_1 = \frac{kT_e}{e}\left[\pm\frac{1-\varepsilon/\theta}{1+\varepsilon}\ln\frac{1}{N_s} + \frac{(1+\theta)\varepsilon j}{\theta j_{i*}}\ln\frac{1+N_s}{2N_s}\right] \tag{10}$$

Let us examine solution (7)-(9). We set $N=N_s$ in (7), and we assume that the charged-particle density in the plasma near the electrode surface is small in comparison with that in the unperturbed plasma, so we have $N_s^2 \ll 1$. We thus find that for a specified current density j the charged-particle fluxes in the space-charged sheath are essentially constant,

$$j_{es} = \frac{\mp j_{i*}(1-N_s^2)+j}{1+\varepsilon} \approx \frac{\mp j_{i*}+j}{1+\varepsilon} = \text{const} \tag{11}$$

$$j_{is} = \frac{\pm j_{i*}(1-N_s^2)+\varepsilon j}{1+\varepsilon} \approx \frac{\pm j_{i*}+\varepsilon j}{1+\varepsilon} = \text{const} \tag{12}$$

Turning now from the solution in the space-charge sheath, we consider relations (11), (12), which in this case hold right up to the electrode surface. Since the mechanism for current transport between the electrode and the plasma depends on the physical properties of the electrode material, on the electrode temperature, and on the conditions near the surface [1], we see that these relations are not meaningful at all current densities. To determine the range of applicability of these relations, we must analyze the physical mechanisms for current transport at the electrode surface. We turn briefly to the emission characteristics of the electrode material.

The emission properties of the electrode material are usually characterized by two parameters: the Richardson constant A and the work function φ_w. These parameters are related to the maximum thermionic current density by the Richardson equation,

$$j_{ew}^{\circ} = AT_w^2 \exp\left(-\frac{e\varphi_w}{kT_w}\right) \tag{13}$$

When there is an accelerating electric field near the electrode surface, the emission current density increases because of the lowering of the potential barrier (the Schottky effect) and is given by

$$j_{ew} = j_{ew}^{\circ}\exp\left[\frac{e^{3/2}E_w^{1/2}}{(4\pi\varepsilon_0)^{1/2}kT_w}\right] \tag{14}$$

The ion current from the electrode surface is related to surface ionization [11] and can be calculated from the Saha-Langmuir equation if the surface properties are known.

However, if the electrode is bounded by a gas containing an alkali vapor, the emission properties of the surface may be significantly changed, to a degree which depends on the surface coverage by the film of alkali atoms. Since there has actually been no study of the effect of an alkali vapor on the work function of an electrode material under conditions approximating those in an MHD installation for the case of various types of impurities in the gas, we will assume here that the electrode surface is characterized at a given temperature by certain emission currents j_{ew}° and j_{iw}, to be determined empirically.

Using Eqs. (11) and (12) with the specified values of j_{ew}° and j_{iw}, we can determine the current-density ranges in which solution (7)-(10) is meaningful. From (12) we see that with $j > \varepsilon^{-1}j_{i*}(1-N_s^2)$ the ion current near the anode changes direction and flows from the anode toward the plasma (and the anode potential drop rises). However, since the ion current from the anode cannot exceed the value j_{iw} according to this theory, Eq. (12) gives us the following limitation on the current density:

$$j \leqslant \frac{j_{iw}(1+\varepsilon)+j_{i*}(1-N_s^2)}{\varepsilon} < \frac{j_{iw}(1+\varepsilon)+j_{i*}}{\varepsilon} = j_a \tag{15}$$

To exceed the value j_a we must also take into account processes such as collisional ionization near the anode and the increase in the ion emission current due to the surface heating. These processes require an additional expenditure of energy and are related to an increase in the potential drop near the anode.

If, within the framework of diffusion theory, we neglect the Schottky effect, we also find a restriction on the current density at the cathode: from Eq. (11) we see that for $j > j_{i*}(1-N_s^2)$ the electron current near

the cathode changes direction and flows from the electrode toward the plasma. Using a maximum electron emission current of $j_e = j_{ew}^\circ$, we find from Eq. (11) the following restriction on the current density:

$$j \leqslant j_{ew}^\circ (1+\varepsilon) + j_{i*}(1 - N_s^2) < j_{e\Sigma}^\circ (1+\varepsilon) + j_{i*} = j_k \tag{16}$$

This type of restriction on the current density near the cathode has been discussed elsewhere [1]. If j_k is to be exceeded, charge carriers must be created near the electrode, through collisional ionization, surface heating, the Schottky effect, etc. These processes require energy and are related to an increase in the potential drop near the cathode.

Inequalities (15) and (16) define the current-density range over which the discharge near the electrode is a distributed discharge; if these inequalities are not satisfied, arcing may occur, and cathode (or anode) spots may arise. There is an important distinction between the conditions for the existence of cathode and anode potential drops: according to (15), (16), collisional ionization or any other process which increases the ion flux is a factor of $1/\varepsilon$ more efficient near the anode than near the cathode.

To calculate the characteristics of the ambipolar-diffusion region from (7)-(10) we must determine the quantity N_s, which depends on both the densities N_{ew} and N_{iw} near the electrode surface and on the solution in the space-charge sheath. In turn, N_{ew} and N_{iw} can be determined if we know the particle fluxes from the electrode surface, which generally depend on the field intensity E_w at the electrode surface and the fluxes in the plasma at the boundary of the sheath, which depend on N_s. Accordingly, N_s, N_{ew}, and N_{iw} must be determined jointly. If we have $N_s^2 \ll 1$, but E_w is not large, so that we have $j_{ew} = j_{ew}^\circ$, the problem can be split into two parts: First, neglecting the term on the order of N_s^2 in comparison with unity, we can determine N_{ew} and N_{iw}, and then we can find N_s.

Let us find the limiting values of the charged-particle densities N_{ew} and N_{iw} in the diffusion approximation. The total number of charged particles which intersect unit area per unit time in the positive (or negative) direction in the gas is given by [12]

$$f_{m\pm} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\mp v_m}^{\infty} (w_m \pm v_m)\, g(w_m)\, dw_m \tag{17}$$

where $g(w_m)$ is the particle distribution function, w_m is the thermal velocity, v_m is the diffusion velocity, and m = e or i.

Assuming the charged particles to have Maxwell distribution of diffusion velocities, we find

$$f_{m\pm} = \frac{n_m w_m}{4} \exp\left[-\frac{4}{\pi}\left(\frac{v_m}{w_m}\right)^2\right] \pm \frac{f_m}{2}\left[1 \pm \Phi\left(\sqrt{\frac{4}{\pi}}\frac{v_m}{w_m}\right)\right], \tag{18}$$

where

$$\Phi(z) = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-t^2)\, dt$$

is the probability integral.

The total number of charged particles emitted by the electrode and intersecting unit area in the positive (or negative) direction is governed by the emission currents:

$$f_{m\pm} = \frac{j_{mw}}{e} \tag{19}$$

From the charged-particle balance at the surface of each electrode,

$$f_m = f_{m+} - f_{m-} \tag{20}$$

we find, using (18) and (19), the charged-particle densities at the electrode surface,

$$\frac{1}{N_{mw}} = \frac{j_{m\infty}}{j_{mw}}\{\exp(-z^2) \pm z\sqrt{\pi}\,[1 \pm \Phi(z)]\}$$
$$z = \frac{1}{2\sqrt{\pi}}\left(\frac{j_{ms}}{j_{m\infty}}\right)\frac{1}{N_{mw}}, \qquad j_{m\infty} = \frac{e n_{m\infty} w_m}{4} \tag{21}$$

where $j_{m\infty}$ is the electron or ion random current in the unperturbed plasma. With $j_{ms} \ll j_{mw}$, we find from (21) the approximate relation [10]

$$\frac{1}{N_{mw}} = \frac{j_{m\infty}}{j_{mw} \mp {}^1/_2 j_{ms}} \tag{22}$$

To evaluate the conditions in the space-charge sheath we must solve system (1), (3) for the specified values of j_i and j_e [determined from (11), (12)] and specified values of N_{ew} and N_{iw} [calculated from Eq. (21)], specifying as a boundary condition at the outer boundary the condition for matching with the ambipolar-diffusion region. In general system (1), (3) must be integrated numerically.

There is one important case, however, in which the calculation can be carried out for the electrode region in a much simpler manner. If the diffusive fluxes of charged particles in the space-charge sheath are small in comparison with the corresponding random fluxes, the particle distribution near the electrode differs little from an equilibrium distribution in an electric field, and we can assume*

$$N_e = N_{ew} \exp(eV / kT_e), \qquad N_i = N_{iw} \exp(-eV / kT_g) \tag{23}$$

where V is the electric potential with respect to the electrode surface. At the outer boundary of the sheath we set $N_{es} = N_{is} = N_s$ and $V = V_s$, where V_s is the potential change in the sheath; from Eqs. (23) we find

$$N_s = N_{ew}^{\vartheta\theta} N_{iw}^{\vartheta}, \qquad V_s = \frac{kT_e}{e} \ln \frac{N_s}{N_{ew}} \qquad \left(\vartheta = \frac{1}{1+\theta}\right) \tag{24}$$

Combining this last expression for V_s with Eq. (10), we find the following expression for the total potential drop near the electrode, for the case $\varepsilon \ll 1$:

$$\Delta V = \frac{kT_e}{e}\left[\pm \ln \frac{1}{N_{ew}} + \frac{(1+\theta)\varepsilon j}{\theta j_{i*}} \ln \frac{1 + N_{ew}^{\vartheta\theta} N_{iw}^{\vartheta}}{2 N_{ew}^{\vartheta\theta} N_{iw}^{\vartheta}}\right] \tag{25}$$

The quantity N_{ew} which appears in Eq. (25) depends on the thermionic current density and thus on the electric field at the electrode surface, so we must in general determine E_w in order to calculate ΔV. A relation between E_w and the field intensity $E_s(N_s)$ can be found from Eqs. (3) and (23),

$$E_w^2 = E_s^2 + 2\left(\frac{n_{e\infty} kT_g}{\varepsilon_0}\right)[\theta N_{iw} + N_{ew} - (1+\theta) N_{ew}^{\vartheta\theta} N_{iw}^{\vartheta}] \tag{26}$$

System (4), (5), (7), (8), (14), (21), (24)-(26) of algebraic and transcendental equations is used to calculate the potential drop near the electrode as a function of the current density, gas parameters, and electrode characteristics.

Using Eqs. (21) and (25) we can determine the physical meaning of restrictions (15) and (16): if we neglect the Schottky effect, we have $N_{ew} \to 0$ as $j \to j_k$, and the potential drop near the cathode is $\Delta V \to \infty$; with $j \to j_a$ the potential drop near the anode becomes infinite.

Calculation for the electrode region simplifies in the case of an equilibrium plasma ($T_e = T_g$): since the plasma parameters do not depend on the current density, and we do not need to use Eq. (5), the form of the other relations in this system simplifies. In particular, Eq. (25) becomes

$$\Delta V = \frac{kT_g}{e}\left[\ln \frac{1}{N_{ew}} + \frac{2\varepsilon j}{j_{i*}} \ln \frac{1 + (N_{ew} N_{iw})^{1/2}}{2 (N_{ew} N_{iw})^{1/2}}\right] \tag{27}$$

Using Eq. (27) along with Eq. (22), we can find the resistance of the electrode region:

$$R_0 = \frac{kT_g}{e}\left\{\frac{1}{2j_{ew} + j_{i*}} + \frac{2\varepsilon}{j_{i*}} \ln\left[\frac{1}{2} + \left(\frac{j_{e\infty} j_{i\infty}}{(2j_{ew} + j_{i*})(2j_{iw} + j_{i*})}\right)^{1/2}\right]\right\} \tag{28}$$

To carry out a calculation for the electrode region by this procedure we must know the gas parameters and electrode characteristics. As was mentioned above, the characteristics of an electrode bounded by a gas containing an alkali admixture may be markedly changed. We will therefore determine these characteristics from experimental data.

---

*Analysis shows that Eqs. (23) hold under the conditions

$$\frac{j_{es} h}{j_{ew} \lambda_e} \ll 1, \qquad \frac{j_{is} h}{j_{iw} \lambda_i} \ll 1 \qquad \left(h = \left(\frac{\varepsilon_0 kT_e}{e^2 n_{e\infty}}\right)^{1/2}\right)$$

where λ_e and λ_i are the electron and ion mean free paths.

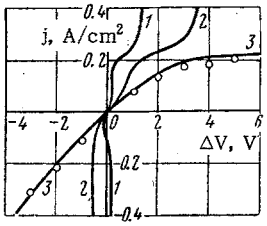

Fig. 1

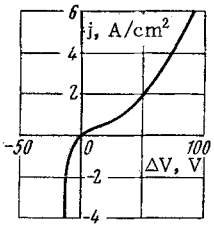

Fig. 2

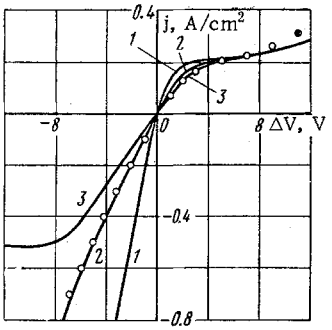

Fig. 3

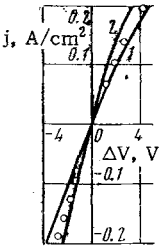

Fig. 4

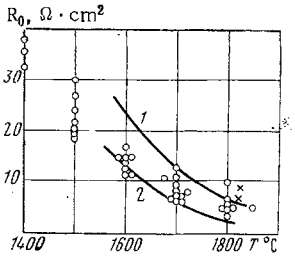

Fig. 5

We use the results of [13], which contains the most extensive information about the experimental conditions. Figure 1 shows the experimental points on the initial region of the current-voltage characteristics of the gas-filled gap between an electrode and a probe in the immediate vicinity (~5 mm) of electrode surface, determined during the flow of current through argon containing 0.15% potassium at $T_g = T_w = 1700°K$.

If, on the basis of theoretical considerations ($\Delta V \to \infty$ as $j \to j_k$), we assume that there is a bend in the characteristic in the cathode ($j > 0$) region at $j = j_k$, we find $j_{ew} = 0.18$ A/cm^2 under these experimental conditions. Since the ion current from the electrode surface is unknown under these experimental conditions, the value of j_{iw} is used as a parameter in the calculations and is determined from the conditions for matching the experimental and calculated characteristics. The physical constants required for the calculation (the collision cross sections, etc.) were taken from the data in [6, 14, 15]. We note that in constructing the current-voltage characteristic of a gas-filled gap between an electrode and a probe the curve calculated from Eq. (25) must be displaced by the magnitude ΔW of the contact potential difference for the electrode-plasma-probe system [if the electrode and probe have identical properties, the characteristic passes through the origin, and we have $\Delta W = \Delta V$ ($j = 0$)].

Before comparing the experimental and theoretical data, we note that, according to [14], the Saha equation with an electron temperature determined from Eq. (8), with δ equal to the energy-loss coefficient for electrons in elastic collisions ($\delta \approx 2$), is a good approximation for small systems only under the conditions $n_e > 10^{14}$ cm^{-3} and $T_e > 3500°K$. For large systems (L ~10 cm) the lower applicability limit of the Saha equation for $T = T_e$ shifts to temperatures on the order of 2300°K. As the temperature or electron density is reduced, the ionization becomes less than that predicted by Eqs. (4) and (5) with $\delta = 2$, and for sufficiently small systems and for a sufficiently low electron density, the gas temperature must play a role in Saha equation (4). Nevertheless, even under these conditions we can use Eqs. (4) and (5) to calculate T_e and $n_{e\infty}$, choosing an effective loss coefficient δ from the condition for matching the calculated and experimental data. The values of δ differ for different experimental conditions (from $\delta \approx 2$ to $\delta \sim 10^2$). At a low current density ($|j| < 0.2$ A/cm^2) under the experimental conditions of [13] ($T_g \approx 2000°K$, $n_e \approx 3 \cdot 10^{12}$ cm^{-3}), we would expect the selective heating of electrons to be negligible.

Comparison of experimental and calculated data confirms this conclusion. Figure 1 shows the calculated "electrode-probe characteristic" for a mixture of Ar + 0.15% potassium with $j_{ew} = 0.18$ A/cm^2, $j_{iw} = 2.5 \cdot 10^{-4}$ A/cm^2 and $T_g = 1950°K$. (Since the experimental gas temperature was measured within about 1-2%, the value of T_g was chosen for a best fit of the calculated and experimental curves, with T_g varied over a range of ±25°K.) Curves 1-3 in Fig. 1 correspond to the values $\delta = 2.66$, 100, ∞. The experimental points are seen to lie close to the calculated points in the case $\delta = \infty$ ($T_e = T_g$).

At sufficiently high current densities ($|j| > 1$ A/cm^2; Fig. 2) there is apparently an appreciable selective heating of electrons. The nonequilibrium ionization can be inferred from the bend of the current-voltage characteristic toward the j axis; this feature shows that the plasma parameters depend on the current density (curves 1 and 2 in Fig. 1). We see that the $\Delta V(j)$ dependence for $|j| > 1$ A/cm^2 in Fig. 2 is similar to curve 2 in Fig. 1 for $|j| > 0.2$ A/cm^2. However, calculation of the characteristic at a high current density goes beyond the framework of the diffusion approximation [5] because of the high electric fields at the electrode ($E_w \sim 10^5$ V/cm).

Figure 3 illustrates the effect of the ion current j_w from the surface on the form of the characteristic; curves 1-3 correspond to ion current densities of $j_w = 5 \cdot 10^{-3}$, $2.5 \cdot 10^{-4}$, and $5 \cdot 10^{-5}$ A/cm^2. From the condition for matching the experimental and calculated characteristics we find $j_{iw} = 2.5 \cdot 10^{-4}$ A/cm^2 under these experimental conditions.

To check the theory we carried out an analogous calculation for the case $T_g = 1600$°C (Fig. 4), determining j_{ew} from the experimental characteristic ($j_{ew} = 0.25$ A/cm^2), and assuming j_{iw} equal to that found earlier, $j_{iw} = 2.5 \cdot 10^{-4}$ A/cm^2. Curves 1 and 2 in Fig. 4 correspond to $T_g = 1850$°K and $T_g = 1875$°K, respectively. We see that the experimental and calculated characteristics agree, within the possible experimental error.

We also calculated the resistance of the electrode region for various surface temperatures. A slight dependence of j_{ew} on the electrode temperature was noted in [13]: as the electrode temperature was varied over the range 1400-1800°C, the j_{ew} value was (on the average) approximately equal to $j_{ew} = 0.1$ A/cm^2. The actual values were scattered over the range 0.05-0.2 A/cm^2 as a result of uncontrollable experimental conditions. These values were also used in the calculations. The j_{iw} value was assumed fixed and equal to the value $j_{iw} = 2.5 \cdot 10^{-4}$ A/cm^2 determined previously, although j_{iw} could in general also change.

Results calculated from Eq. (28) are shown in Fig. 5; curve 1 corresponds to the value $j_{ew} = 0.05$ A/cm^2, and curve 2 corresponds to $j_{ew} = 0.2$ A/cm^2. Over the range 1600-1800°C the experimental points lie within the range of the expected scatter. For $T_g < 1600$°C, the size of the perturbed-plasma region is comparable to the electrode size, so the resistance calculated from Eq. (28) is too high.

In conclusion we will discuss the conditions under which this theory is applicable. If the electron temperature is known (e.g., experimentally) the limitations on the theory are related to the general conditions for the applicability of diffusion equations with constant coefficients [5] and the possible use of Eqs. (23) in the space-charge sheath.

We note that Eq. (26), obtained with the help of (23), does not reflect the tendency of the space-charge sheath to expand (albeit slowly) at high applied voltages, so calcuations carried out for the cathode region under the condition $j > j_k$ yield values of ΔV which are too low. When the electron temperature is determined from Eqs. (4) and (5), restrictions associated with their applicability also hold [14]. In an equilibrium plasma the restrictions are less severe. In this case the basic contribution to the potential drop near an electrode comes from the second term in Eq. (25), and the error associated with the approximate determination of N_s has only a slight effect on the calculated results.

The authors thank G. A. Lyubimov for interest in the study and for discussion of the results, and B. V. Parfenov for graciously furnishing the necessary experimental data from [13].

LITERATURE CITED

1. G. A. Lyubimov, "Electrode layers in which there is a rapid potential change near a 'hot' electrode," Teplofiz. Vys. Temp., 4, No. 1 (1966).
2. A. Pinchak and É. Zhukoski, "Volume and surface phenomena in a plasma containing admixtures," in: Applied Magnetohydrodynamics [in Russian], Moscow, Mir (1965).
3. L. B. Loev, Fundamental Processes of Electrical Discharge in Gases, Wiley (1939).
4. L. E. Kalikhman, Elements of Magnetohydrodynamics [in Russian], Moscow, Atomizdat (1964).
5. É. Blyu, D. Ingol'd, and V. Ozerov, "Diffusion of electrons and ions in a neutral gas," Thermionic Energy Conversion [in Russian], Vol. 2, Moscow, Atomizdat (1965).
6. T. Hiramoto, "Nonequilibrium characteristics of working plasma for magnetoplasmadynamic (MPD) generators," J. Phys. Soc. Japan, 20, No. 6 (1965).
7. L. M. Biberman, V. S. Vorob'ev, and I. T. Yakubov, "Ionization and recombination functions in a plasma," Teplofiz. Vys. Temp., 7, No. 4 (1969).
8. J. L. Kerrebrock, Proceedings of the Second Symposium on the Engineering Aspects of MHD, Philadelphia (1961).
9. J. L. Kerrebrock, "Nonequilibrium ionization due to electron heating. Theory," AIAA Journal, 2, No. 8, 1072-1080 (1964).
10. G. A. Lyubimov and V. N. Mikhailov, "Analysis of the perturbed-plasma region near an electrode," Izv. Akad. Nauk SSSR, Mekhan. Zhidk. i Gaza, No. 3 (1968).
11. É. L. Zandberg and N. I. Ionov, Surface Ionization [in Russian], Nauka, Moscow (1969).

12. G. N. Patterson, Molecular Flow of Gases [in Russian], Fizmatgiz, Moscow (1960).
13. V. O. German, G. A. Lyubimov, and B. V. Parfenov, "Potential drop near a molybdenum electrode in an argon flow with a potassium admixture," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 4 (1970).
14. K. M. Aref'ev and I. I. Paleev, Fundamentals of Thermionic and Magnetohydrodynamic Energy Conversion [in Russian], Atomizdat, Moscow (1970).
15. G. W. C. Kaye and T. H. Laby, Tables of Physical and Chemical Constants and Some Mathematical Functions, Longmans, Dorchester, England (1959).